WORKSHOP ON MODEL THEORY OF FINITE AND PSEUDOFINITE STRUCTURES

UNIVERSITY OF LEEDS
JULY 27-29

Wednesday, July 27th

10:00 - 10:40 **Welcoming and Coffee.**

10:40 - 11:40 **Dimension and dividing in generalised measurable structures**
Sylvy Anscombe - University of Central Lancashire

The notion of a 'measurable structure' was introduced by Macpherson and Steinhorn in [2]. In ongoing joint work between the speaker, Macpherson, Steinhorn, and Wolf ([1]), we propose the broader framework of 'generalised measurable structures'. Both the new definition and the original aim to capture the idea of a structure in which families of definable sets may be assigned a 'measure' and 'dimension' in a nearly-uniform way.

In measurable structures, dimensions are always natural numbers; but in generalised measurable structures, dimensions may be much more complicated. For example, we do not require dimension to be well-founded!

In this talk I will explore the link between dimension, dividing (of formulas, in the sense of stability theory), and D-rank in (generalised) measurable structures. I will also present some examples which show that generalised measurable structures may be far from the supersimple.

[1] Sylvy Anscombe, Dugald Macpherson, Charles Steinhorn, and Daniel Wolf. *Multidimensional Asymptotic Classes.* Manuscript, 2016.

[2] Dugald Macpherson and Charles Steinhorn. *One-dimensional asymptotic classes of finite structures.* Trans. Amer. Math. Soc., 360:411Ð448, 2008.

11:45 - 12:45 **Metrically homogeneous graphs of infinite diameter**
Gregory Cherlin - Rutgers University

A connected graph is metrically homogeneous if the associated metric space (under the graph metric) is homogeneous. We discuss a conjectured classification of these graphs, with particular attention to the following point: if the classification is correct in the case of bounded diameter, then it is also correct in the case of infinite diameter.

In particular, it makes sense in principle to approach this classification problem via induction on the diameter.

12:45 - 14:20 **Lunch Break**

14:20 - 15:20 **Fractional Helly property in model theory**
Artem Chernikov - UCLA

Classical Helly's theorem says that given finitely many convex sets in R^d, if any $d+1$ of them have a non-empty intersection, then the whole family has a non-empty intersection. A more recent fractional version of Katchalski and Liu says that if the assumption holds for the α-fraction of all

$d+1$ tuples, then the conclusion holds for a β-fraction of the whole family (and β depends only on α and d). Matousek had demonstrated that this fractional Helly property, or FHP, also holds for families of finite VC-dimension (e.g. for semialgebraic sets of bounded description complexity). In this talk we consider FHP and its variants in the model theoretic setting, for definable families of sets relative to certain finitely additive measures, and its relation to Shelah's classification, forking and burden.

15:25 - 15:55 **Embedded asymptotic classes**
Gwyneth Harrison-Shermoen - University of Leeds

A (one-, N-, or multidimensional) *asymptotic class* (defined, respectively, by MacPherson and Steinhorn (2008), Elwes (2007), and Anscombe, MacPherson, Steinhorn, and Wolf (in preparation)) is a class of finite structures with uniform bounds on the sizes of definable sets. We will discuss some of the background definitions and results of the theory of (various types of) asymptotic classes before introducing the definition of an *embedded asymptotic class*: roughly, a class of pairs $(\mathcal{M}, A)$ - where $\mathcal{M}$ is an infinite structure and A is finite substructure of $\mathcal{M}$ - with uniform bounds on the sizes of M-definable subsets of A. We will present the initial results and examples of embedded asymptotic classes. This is a work in progress, joint with Dugald MacPherson.

15:55 - 16:35 **Coffee Break**

16:35 - 17:20 **An introduction to relational complexity: background, questions, and a few answers**
Joshua Wiscons - California State University, Sacramento

The relational complexity of a structure $\mathbf{X}$ is the least k (if one exists) for which the orbits of $\mathrm{Aut}(\mathbf{X})$ on X^k "determine" the orbits of $\mathrm{Aut}(\mathbf{X})$ on X^n for all $n < \omega$. This invariant originated in Lachlan's classification theory for homogeneous finite—and more generally, countable stable—relational structures, but not much was known about the complexities of specific structures until the work of Cherlin, Martin, and Saracino in the 1990's.

This talk will begin from first principles with a focus on how to compute (or at least bound) the relational complexities of a handful of familiar structures. Following this, the goals are to present a few general open problems about the invariant, including Cherlin's conjecture for finite primitive structures of complexity 2, and discuss recent progress on them. Parts of the talk will use methods of finite permutation group theory.

Thursday, July 28th

09:00 - 10:00 **Amalgamation and symmetries in the finite**
Martin Otto - Technische Universitat Darmstadt

Consider an amalgamation task for finite structures of the following kind: we seek a finite relational structure that is consistent with a given atlas of finite relational structures (as local charts for the desired amalgam) and partial isomorphisms between them (as changes of co-ordinates between charts). We aim for generic, locally free, finite solutions that do not break any symmetries of the specification. Our construction involves suitable groupoids (or inverse semigroups) whose Cayley graphs provide the backbones for the global amalgamation pattern. Model-theoretic applications concern new perspectives on the finite model property for certain fragments of first-order logic and - closely linked - extension properties for partial isomorphisms, in particular a new proof and apparent strengthening of a theorem of Herwig and Lascar concerning the lifting of local symmetries to global automorphisms in finite structures.

10:00 - 10:40 **Coffee Break**

10:40 - 11:40 **Simple homogeneous structures**
Vera Koponen - Uppsala University

All binary simple homogeneous structures have finite SU-rank (hence supersimple) and can be classified up to certain "finite blocks". (A complete classification would require a classification of all finite binary homogeneous structures.) I will explain this. The other part of my talk is, I guess, to state problems regarding nonbinary simple homogeneous structures, give some examples and to try to outline some directions in which one could try to make progress in this direction.

11:45 - 12:15 **Asymptotic classes of residue rings**
Ricardo Bello Aguirre - University of Leeds

It follows from previous work that ultraproducts of a certain class of finite residue rings are supersimple. I will present some work in progress on showing this class is an asymptotic class.

12:20 - 12:50 **On alternatives for pseudofinite groups**
Françoise Point - Université Paris Diderot - Paris 7

We investigate alternatives for pseudofinite groups of the same character as the Tits alternative for linear groups. First, we will recall various notions of approximability of a group by a class of finite groups and definability results in the class of pseudofinite groups. Then using former results of S. Black and G. Traustason, we derive structural consequences for pseudofinite groups that do not contain a free group, respectively a free monoid, on two generators. This is a joint work with A. Ould Houcine.

12:50 - 14:20 **Lunch Break**

14:20 - 15:20 **Structure and enumeration theorems in hereditary properties of $\mathcal{L}$-structures**
Caroline Terry - University of Illinois - Chicago

The study of structure and enumeration for hereditary graph properties has been a major area of research in extremal combinatorics. Over the years such results have been extended to many combinatorial structures other than graphs. This line of research has developed an informal strategy for how to prove these results in various settings. In this talk we formalize this strategy. In particular, we generalize certain definitions, tools, and theorems which appear commonly in approximate structure and enumeration theorems in extremal combinatorics. Our results apply to classes of finite $\mathcal{L}$-structures which are closed under isomorphism and model-theoretic substructure, where $\mathcal{L}$ is any finite relational language with maximum arity at least two.

15:25 - 15:55 **Extending partial isometries in metric spaces with forbidden subspaces**
Gabriel Conant - University of Notre Dame

A class $\mathcal{K}$ of finite structures is said to have the Hrushovski property if, for every A in K there is some B in K such that A is a substructure of B and any partial automorphism of A extends to a total automorphism of B. When $\mathcal{K}$ is a Fraïssé class, the Hrushovski property often leads to interesting topological properties of the automorphism group of the Fraïssé limit. We present two theorems concerning the Hrushovski property in the context of binary relational structures, especially graphs and metric spaces. Our starting point is Solecki's proof of the Hrushovski property for finite metric spaces. We generalize this argument, and combine it with an inductive construction, to prove the Hrushovski property for certain classes of finite generalized metric spaces. This includes many known examples (graphs, metric spaces, and ultrametric spaces), along with some new ones, such as metric spaces with restricted distance sets studied by Delhommé, Laflamme, Pouzet, and Sauer. We then consider metric spaces with forbidden configurations, such as those arising from graphs omitting cycles of odd length, developed by Komjáth, Meckler, and Pach. Our second theorem analyzes extension of isometries in classes of metric spaces omitting some suitable family of forbidden subspaces. As a corollary we obtain the Hrushovski property for some new examples, such as the aforementioned graphs omitting odd cycles, as well as other so-called Urysohn-Henson hybrids coming from Cherlin's catalog of metrically homogeneous graphs.

15:55 - 16:35 **Coffee Break**

16:35 - 17:20 **Metric ultraproducts of finite metric groups**
Aleksander Ivanov - University of Wroclaw

Let us consider the class $\mathcal{G}$ of all continuous structures which are metric groups (G, d) with bi-invariant metrics $d \leq 1$. Let $\mathcal{G}_{sof} \subset \mathcal{G}$ be the subclass of all closed metric subgroups of metric ultraproducts of finite symmetric groups with Hamming metrics. We call metric groups from $\mathcal{G}_{sof}$ *sofic metric groups*. The class $\mathcal{G}_{w.sof}$ of weakly sofic continuous metric groups, consists of continuous metric groups (G, d) which embed into metric ultraproducts of finite metric groups with invariant metrics bounded by 1. In a similar way we define the classes $\mathcal{G}_{l.sof}$ and $\mathcal{G}_{hyplin}$ of continuous metric groups which are linear sofic and hyperlinear as metric groups.

We study relationships among the classes of the collection

$$\{\mathcal{G}, \mathcal{G}_{sof}, \mathcal{G}_{w.sof}, \mathcal{G}_{hyplin}, \mathcal{G}_{l.sof}\}.$$

All of them are axiomatizable in continuous logic. It is clear that $\mathcal{G}_{sof} \subseteq \mathcal{G}_{w.sof} \subseteq \mathcal{G}$. Moreover, by some arguments of Arzhantseva and Päunescu $\mathcal{G}_{l.sof} \subseteq \mathcal{G}_{w.sof}$. We show that the class $\mathcal{G}_{w.sof} \setminus (\mathcal{G}_{sof} \cup \mathcal{G}_{hyplin} \cup \mathcal{G}_{l.sof})$ is not empty. We emphasize that in these classes groups are considered together with metrics. Thus Gromov's problem if any group is sofic still remains open.

We connect these questions with the topic of metric transformations of metric spaces (initiated by I.J.Schoenberg and W.A.Wilson in the 1930s).

I am going to present some concrete examples which are principal in our approach.

17:25 - 17:55 **$>k$-homogeneous graphs and homogenizable structures**
Ove Ahlman - Uppsala University

A structure $\mathcal{M}$ over a finite relational language is called homogeneous ($>k$-homogeneous) if for each finite $\mathcal{A} \subseteq \mathcal{M}$ ($|\mathcal{A}| > k$) and embedding $f : \mathcal{A} \to \mathcal{M}$, f may be extended into an automorphism. A structure $\mathcal{N}$ is homogenizable if there are formulas $\varphi_1(\overline{x}_1), \ldots, \varphi_n(\overline{x}_n)$ such that if the sets defined in $\mathcal{N}$ are added as relations in $\mathcal{N}$, then the structure created is homogenous.

The $>k$-homogeneous structures are in some sense as close to being homogeneous we can get without actually being homogeneous, and clearly if such a structure is ω-categorical it is also homogenizable. In this talk I will give some thoughts on this problem and show an explicit characterization of the $>k$-homogeneous infinite graphs. I will also, in higher generality, discuss the homogenizable structures. Just like the homogeneous structures have a strong connection to its set of finite substructures and embedding-extensions there is also a similar correspondence when it comes to homogenizable structures. I will give an approach to study the homogenizable structures which focus more on the properties of the structures themselves rather than their ages.

Friday, July 29th

09:00 - 10:00 **Determining finite simple images of finitely presented groups**
David Evans - Imperial College

I will discuss on-going joint work with Martin Bridson and Martin Liebeck which addresses the question: for which collections of finite simple groups does there exist an algorithm that determines the images of an arbitrary finitely presented group that lie in the collection?

10:00 - 10:40 **Coffee Break**

10:40 - 11:40 **Some sufficient conditions for tight control of the asymptotics of definable sets.**
Cameron Hill - Wesleyan University

I will discuss some sufficient conditions for (minor generalizations of) MS-measurability and "asymptoticness" of classes of finite structures. These conditions come from combinatorial phenomena that are not obviously related to dimension and measure, such as 0,1-laws for first-order logic and structural Ramsey theory.

11:45 - 12:15 **Invariant measures via finite structures**
Cameron Freer - Massachusetts Institute of Technology

A random structure with a fixed countably infinite underlying set is said to be *exchangeable* when its joint distribution is invariant under permutations of the underlying set; such a distribution is invariant under the logic action. In joint work with Ackerman and Patel, we provide a model-theoretic characterization of those countably infinite structures admitting exchangeable constructions. We also address related questions, such as the number of ergodic, or of properly ergodic, invariant measures concentrated on the models of a given theory. Joint work with Nathanael Ackerman, Alex Kruckman, Aleksandra Kwiatkowska, Jaroslav Nesětřil, Rehana Patel, and Jan Reimann.

12:20 - 12:50 **Infinite limits of other random graphs**
Richard Elwes - University of Leeds

For model theorists (and pure mathematicians generally) a 'random graph' is usually a structure produced by an Erdös-Rényi process, in which every pair of nodes is connected with some fixed probability. The infinite limit of this process is a very well-known countably-categorical structure. Mathematically beautiful although they are, for network-scientists trying to model real-world phenomena such as the worldwide web, Erdös-Rényi graphs are largely inadequate. Various other random graph-growing processes have been developed; chief among them is the Preferential Attachment paradigm of Barabási-Albert. In this talk, we will describe some such processes, and say something about the structures that arise as their infinite limits.

12:50 - 14:20 **Lunch Break**

14:20 - 15:20 **Stable hypergraph regularity**
Rehana Patel - Olin College

We extend results of Malliaris and Shelah on regularity lemmas for stable graphs to the case of hypergraphs. Joint work with Nathanael Ackerman and Cameron Freer.

15:25 - 15:55 **A model for the representation theory of rings of integers.**
Lubna Shaheen - University of Oxford

The aim of this project is to attach a geometric structure to the ring of integers and to understand Spec($\mathbb{Z}$) from the point of view of stability theory. We describe a category of certain representations of integral extensions of $\mathbb{Z}$ and establish its tight connection with the space of elementary theories of pseudo-finite fields. From model-theoretic point of view the category of representations is a multi-sorted structure which we prove to be super-stable with pre-geometry of trivial type. It comes as some surprise that a structure like this can code a rich mathematics of pseudo-finite fields.

15:55 - 16:35 **Coffee Break**

16:35 - 17:05 **Multidimensional exact classes and Lie coordinatisation**
Daniel Wolf - University of Leeds

We introduce the notion of a multidimensional exact class, as jointly developed with Anscombe, Macpherson and Steinhorn. We then sketch a proof of the following result: For any language $\mathcal{L}$ and for any positive $d \in \mathbb{N}$, the class $\mathcal{C}(\mathcal{L}, d)$ of all finite $\mathcal{L}$-structures with at most d 4-types forms a multidimensional exact class. The proof makes extensive use of smooth approximation, a notion introduced by Lachlan in the 1980s, and the equivalent notion of Lie coordinatisation, as deeply developed by Cherlin and Hrushovski in their book *Finite Structures with Few Types* (Princeton, 2003).